Metacognitive Knowledge, Beliefs and Classroom Mathematics

Merrilyn Goos

The University of Queensland

Abstract.

This paper reports on an exploratory study that investigated links between the quality of secondary school students' metacognitive knowledge, their beliefs about mathematics, and their perceptions of school mathematics practice. Two questionnaires were administered to a sample of 170 Year 11 and Year 10 students in three Queensland schools. Analysis of the responses points toward teaching and learning practices associated with empowering beliefs and well developed metacognitive knowledge. In particular, the potential value of mathematical discussion with peers is highlighted.

Theoretical Background

Metacognition, or *knowledge* about and *control* over one's own cognitive processes, is often considered to be critical to effective mathematical thinking and problem solving (Garofalo and Lester, 1985), and the ability to monitor one's problem solving behaviour distinguishes novices from experts in the domain (e.g. Venezky and Bregar, 1988). However, metacognition is not limited to purely cognitive activity, but often interacts with affects and beliefs. For this reason it is helpful to consider metacognition as not only knowledge and control of *process* (strategic knowledge and self-monitoring), but also knowledge and control of *self* (Marzano, Brandt, Hughes, Jones, Presseisen, Rankin and Suhor, 1988). *Knowledge-of-self* includes assessments of one's own mathematical competence and beliefs about the nature of mathematical ability in general, attributional beliefs about the causes of success and failure, awareness of the consequences of affective traits such as motivation, anxiety and perseverance, and awareness of strengths and weaknesses with respect to particular types of tasks. *Control-of-self* entails monitoring and regulating commitment, attitudes and attention in order to maintain task involvement.

While metacognitive processes influence mathematical performance, the way in which students select and deploy metacognitive knowledge and strategies may be in turn be shaped by their *beliefs* about mathematics and how it is learned. In a recent Australian study involving Year 10 students, Stacey (1990) demonstrated that students who valued understanding and were willing to think for themselves had higher scores for metacognitive knowledge and problem solving performance than those who preferred to learn by rote. Attitudes and beliefs influenced the extent to which students made use of their mathematical knowledge: even low achievers who were confident and self-reliant in using simple strategies were more successful in tackling unfamiliar problems than high achievers who were unable, or unwilling, to apply their more extensive knowledge beyond textbook settings.

Cobb (1986) considers that students' beliefs about mathematics are constructed as a response to their previous *classroom experiences*. Two influences are critical: the cultural context within which instruction occurs, and the social interactions between teacher and students. Schooling often devalues the informal, intuitive reasoning used to solve everyday problems and tries to replace it with academic reasoning, while emphasising that academic mathematics must be expressed in the culturally acceptable form. As students try to master the conventions of symbol manipulation, they may come to believe that producing correct form is more important than understanding the subject matter. In addition, teacher-student interactions based on imposition, rather than negotiation, lead students to believe that they must rely on the teacher as the source of all mathematical knowledge.

Under the influence of these classroom contextual factors, students become adept at generating a 'veneer of accomplishment' (Lave, Smith and Butler, 1989, p. 74), because they learn that success in mathematics can be achieved by performing, without necessarily understanding, the tasks set by their teachers. Similarly, Resnick (1989) argues that the common view of mathematics as a well-structured discipline, with a clear hierarchy of knowledge, unambiguous meanings and no open questions, is responsible for students' faulty beliefs. She recommends that mathematics be taught as an ill-structured discipline by encouraging multiple interpretations and more collaborative interactions in order to foster mathematical thinking and a disposition to meaning construction.

Conclusions from the research outlined above could be summarised as follows: as a result of the mathematics instruction they receive in school, students develop beliefs and attitudes that become part of their metacognitive knowledge and influence how they use metacognitive strategies. The study reported here gathered data concerning these relationships.

Aims

The purpose of this pilot study was to trial two questionnaires to be used as pre and post measures in a larger experimental study involving senior secondary school students. (Here, the questionnaires were administered on only one occasion.) The first questionnaire gathered information on students' beliefs about mathematics and their perceptions of school mathematics practices, while the second probed students' metacognitive knowledge. The aims of the study are expressed in the following research questions:

1. To what extent are beliefs about mathematics related to classroom practices?
2. What is the relationship between metacognitive knowledge and beliefs about mathematics?

Method

Three Queensland secondary schools participated in the trialing of questionnaires: a Catholic co-educational college located in a provincial city, a government high school in a middle class outer Brisbane suburb, and an inner Brisbane Catholic girls' school. Seven Year 11 classes and one high ability stream Year 10 class took part, giving a sample of 170 students.

Instruments

The 43 item Beliefs Questionnaire consisted, for most part, of statements to which students were asked to respond on a four or five point Likert scale. Some statements were based on those found in similar instruments used by Clarke, Waywood and Stephens (1993), McDonagh and Clarke (1994), and Schoenfeld (1989), while others were constructed for the purpose of this study. The questionnaire was divided into four sections: the first section dealt with attributions for success and failure in mathematics tests, the second contained statements reflecting beliefs about mathematics, the third sought students' perceptions of school mathematics practices, and the fourth section asked students to provide information on their mathematics achievement level and to rate their own ability and effort compared to classmates.

The free response Self-Knowledge Questionnaire, developed in an earlier study for use with Year 11 students (Goos, 1993), focussed mainly on students' metacognitive knowledge-of-self and knowledge-of-process (Table 1). Some items were drawn from Schoenfeld's (1989) questionnaire, while others were constructed from Garofalo's (1987) suggestions for questions that teachers could put to their students to help develop their awareness.

Table 1 Structure and purpose of Self-Knowledge Questionnaire

Question	Knowledge of ...
1 (a) What kind of mistakes do you usually make in maths?	Self
(b) Why do you think you make those mistakes?	Self
(c) What can you do about them?	Process
2. What do you do when you are stuck on a problem?	Process
3. (a) What kinds of problems are you best at?	Self
(b) Why?	Self
4. (a) What kinds of problems are you worst at?	Self
(b) Why?	Self
(c) What can you do to improve on these?	Process
5. How do you know when you understand something in maths?	Self

Data Coding and Analysis

The first research question was addressed by calculating correlations between items in Section 2 (beliefs about mathematics) and Section 3 (perceptions of school mathematics) of the Beliefs Questionnaire.

Because students gave open ended responses to the Self-Knowledge Questionnaire it was necessary to develop a coding scheme before the data could be analysed. Categories were first created to allow similar responses to be identified and grouped. It then became clear that some students possessed more sophisticated metacognitive knowledge than others. A scoring protocol was therefore devised to measure the *quality* of students' metacognitive self-knowledge.

Two dimensions of metacognitive quality were measured. The SOLO taxonomy (Biggs and Collis, 1982) was used to assess the *structural complexity* of responses by assigning a score of 1 (prestructural) to 5 (extended abstract). Because the wording of Questions 1a, 3a and 4a made it unlikely that they would elicit relational or extended abstract responses, these Questions were not scored. A mean SOLO score for the remainder of the questionnaire was calculated for each student.

The second dimension of quality classified and assessed the *content* of responses by assigning a score on a 3, 4 or 5 point scale, the size of the scale being influenced by the variety of conceptually distinct responses. A score of 1 was awarded for the responses 'don't know', 'nothing', or 'no response', mid-range scores for responses that were general in nature or referred to dependent or ineffective strategic behaviour, and the highest scores for responses that were specific, emphasised understanding, or referred to independent strategic behaviour. The scores for each question were summed for the whole questionnaire, giving each student a total score that could range from a minimum of 10 (ten question parts with a minimum score of 1 for each) to a maximum of 41. The total score was then made independent of the size of the rating scales by converting it to an index of content quality, whose values could range from zero to one, via the following transformation: Content Quality = (Total Score - 10) ÷ 31.

Although high-structure responses also tended to have high quality content ($r = .713$, $p<.001$), a final variable was computed as follows to combine both these qualities in students' responses: Overall Quality = (SOLO mean score) × (Content Quality).

The second research question was then dealt with by calculating correlations between variables created to measure students' responses to the Self-Knowledge Questionnaire and variables derived from each section of the Beliefs Questionnaire. For both research questions, correlations greater than .25 were considered worth reporting.

Results

Research Question 1: To what extent are beliefs about mathematics related to classroom practices?

Only three items in Section 2 of the Beliefs Questionnaire had correlations greater than .25 with any item in Section 3 (Table 2). These related to beliefs that everything in mathematics is already known (Item 16), that students can experience personal creativity and discovery in mathematics (Item 17), and that there is only one correct way to solve mathematics problems (Item 18).

Table 2 Correlations between Beliefs and Perceptions of School Mathematics Practices

	Section 2 (Beliefs)		
	16. Everything known	17. Creativity, discovery	18. One way to solve
Section 3 (Perceptions of School Practice)			
When the teacher asks a question during a maths lesson ...			
26. There are lots of possible correct answers you might give.	-.152*	.252***	-.110
When I get the wrong answer to a maths problem ...			
30. I try to work out for myself where I went wrong.	-.323***	.247***	.258***
When I'm doing maths at school, I'm likely to be ...			
34. Talking about maths to other students.	-.285***	.204**	-.208**

Notes. 1. *p<.05, **p<.01, ***p≤.001 2. Correlations greater than .25 are underlined.

In general, students who saw mathematics as a closed, well-structured discipline (as expressed by responses to Items 16, 17 and 18) were less likely to accept the possibility of there being many correct answers to teacher questions (Item 26), less independent in trying to locate and correct their errors (Item 30), and less likely to spend time talking about maths to other students (Item 34). This may indicate that such students did not see themselves, or their peers, as sources of mathematical knowledge or authority.

Research Question 2: What is the relationship between metacognitive knowledge and beliefs about mathematics?

Information on the relationships between self-knowledge and beliefs was obtained by calculating correlations between the composite variables measuring overall quality of metacognitive knowledge and the variables obtained from the Beliefs Questionnaire. Correlations greater than .25 are reported in Table 4.

Table 4. Correlations between Metacognitive Knowledge (Composite Scores) and Beliefs

	Metacognitive Knowledge Score		
	Structural Complexity	Content Quality	Overall Quality
Section 1 (Attributions)			
When I do well in a maths test ...			
4. It's because I'm good at maths.	-.257***	-.275***	-.289***
When I do badly in a maths test ...			
8. It's because the teacher doesn't like me.	.290***	.301***	.292***
10. It's because I'm no good at maths.	.175*	.301***	.245**
Section 2 (Beliefs)			
16. Everything important about mathematics is already known by mathematicians.	.284***	.298***	.328***
21. The ideas of mathematics can only be explained using mathematical language and special terms.	.225**	.214**	.256***
Section 3 (Perceptions of School Practice)			
When I get the wrong answer to a maths problem ...			
30. I try to work out for myself where I went wrong.	-.213**	-.309***	-.278***
When I'm doing maths at school I'm likely to be ...			
34. Talking about maths to other students.	-.269***	-.290***	-.307***

Notes. 1. $^{*}p<.05$, $^{**}p<.01$, $^{***}p\leq.001$ 2. Because of the direction of the scales on the Beliefs Questionnaire, negative correlations indicate a positive association with quality of metacognitive knowledge, and vice versa. 3. Correlations greater than .25 are underlined.

Students with the highest metacognitive scores were more likely than others to believe in their own ability as the cause of their successes in mathematics tests, to try to discover for themselves the sources of their errors, and to spend time in class talking to other students about mathematics. Students with the lowest metacognitive scores were more likely than others to view their own lack of ability, or the teacher's attitude towards them, as the cause of their failures in mathematics tests, and to believe that mathematics is a fixed, rather than evolving, body of knowledge, best communicated by using the technical language peculiar to the discipline.

There is one further correlation of interest to report. The decision to assign a SOLO score to Self-Knowledge Questionnaire responses was prompted by the observation that students varied considerably in the number of strategies they reported for taking action when stuck on a problem (Question 2), and in their ability to describe the circumstances under which they would use those strategies. The SOLO score for this question would therefore indicate the extent to which students had access to a range of strategies and could justify their utility. Correlations were calculated between this score and all the variables derived from the Beliefs Questionnaire. Only one correlation greater than .25 was found: students with the greatest flexibility in dealing with obstacles tended to be those who talked to their peers when doing mathematics at school ($r = -.285$, $p<.001$).

Summary and Conclusions

It has been suggested that beliefs about mathematics are constructed as a result of classroom experience, and that many students separate their conceptions of school mathematics and abstract mathematics. There is value, therefore, in identifying classroom practices that are linked to *positive* beliefs. The findings supported Resnick's (1989) recommendations for teaching mathematics as an ill-structured discipline. Students who associated mathematics with discovery, creativity and multiple solution paths were more likely than others to report school mathematics practices that allowed them to propose and evaluate their own ideas without relying on the teacher as the sole possessor of authentic knowledge.

Associations between the quality of metacognitive knowledge and particular beliefs and activities were also found. Students with the most sophisticated self-knowledge were distinguished firstly by their willingness to take personal responsibility for their successes and failures, and their independence in trying to correct their errors. Belief in the ability to control one's own learning is an important aspect of metacognitive awareness, and is also needed before such control can be exercised as metacognitive self-regulation (Biggs, 1987). A second characteristic of students with high quality metacognitive knowledge was their treatment of mathematics as a participatory, creative endeavour, confirming a connection between metacognition and beliefs about mathematics alluded to by Stacey (1990).

Further Work

This study has found associations between students' metacognitive knowledge, beliefs about mathematics as a discipline, and perceptions of classroom practice that are consistent with arguments advanced in the research literature. Although the correlations were only moderate (and correlation does not imply causality), in an exploratory study such as this they are useful for suggesting questions that could guide future research. One such question concerns the value of peer discussion, as it was found that 'talking to other students about maths' was related to both empowering beliefs and well elaborated metacognitive knowledge. This issue is particularly deserving of further investigation, given the emphasis on collaborative learning in recent Australian curriculum documents (e.g. Australian Education Council, 1991) and current research interest in peer learning (for a review see Davidson and Kroll, 1991).

Other aspects of the pilot study may require improvement or extension in future work. First, the rating scales created to measure quality of metacognitive knowledge were derived from the responses of students in this sample and need to be validated on other students. Second, the Self-Knowledge Questionnaire may provide even more information if used as an interview script so that students could be asked to elaborate on their responses. Another limitation of the present study relates to the nature of the data collected. Self-reports of cognitive and metacognitive phenomena, such as those elicited by the Beliefs and Self-Knowledge Questionnaires, may not correspond to students' actual behaviour and should be verified via other methods of data collection. In the main experimental study, students' reported beliefs, perceptions of classroom practices, and metacognitive knowledge will be checked through classroom observation, interviews, videotaping students' problem solving, and asking students to recall their thought processes as they watch the videotapes. In this way, it is hoped that further insights into the relationships between metacognitive knowledge, mathematical beliefs and classroom practices will be gained.

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